

Dispersion relations and the spin polarizabilities of the nucleon

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Abstract

A forward dispersion calculation is implemented for the spin polarizabilities $\gamma_1, \dots, \gamma_4$ of the proton and the neutron. These polarizabilities are related to the spin structure of the nucleon at low energies and are structure-constants of the Compton scattering amplitude at $\mathcal{O}(\omega^3)$. In the absence of a direct experimental measurement of these quantities, a dispersion calculation serves the purpose of constraining the model building, and of comparing with recent calculations in heavy baryon chiral perturbation theory.

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1. Introduction.

The response of the internal degrees of freedom of the nucleon to an external electromagnetic field can be parametrized in terms of the structure dependent polarizabilities. The classic process for studying such quantities is Compton scattering at energies below the resonance region. In an expansion of the Compton scattering amplitude in powers of the incident photon energy ω , the $\mathcal{O}(1)$ and $\mathcal{O}(\omega)$ terms depend only on the mass M , the electric charge e and the anomalous magnetic moment κ of the nucleon. Therefore, no information on the excitation of the internal degrees of freedom can be obtained up to $\mathcal{O}(\omega)$; it is only at $\mathcal{O}(\omega^2)$ that the amplitude becomes sensitive to the internal excitation of the nucleon. At $\mathcal{O}(\omega^2)$, the amplitude is parametrized in terms of the electric (α) and magnetic (β) polarizabilities [1], which describe the deformation of the system in the presence of a static electric and magnetic field. At $\mathcal{O}(\omega^3)$, as demonstrated recently by Ragusa [2], four new polarizabilities appear, $\gamma_1, \gamma_2, \gamma_3$, and γ_4 , the “spin polarizabilities”. As explained by Ragusa, these can be interpreted as the response to the external fields of a magnet that has an internal structure. For related previous work on the $\mathcal{O}(\omega^3)$ spin structure of the Compton scattering, see Ref. [3].

Contrary to α and β , none of the γ_i ’s has been measured experimentally, although there has been a proposal for that purpose [4]. Only the spin-dependent polarizability for forward scattering involving three of the spin polarizabilities, $\gamma = \gamma_1 - \gamma_2 - 2\gamma_4$, has been experimentally constrained through an analysis involving photoproduction multipoles [5], with the result $\gamma = -1.3$ (here and thereafter we quote all polarizabilities in units of 10^{-4} fm^4). An earlier multipole analysis [6] obtained $\gamma = -1.0$.

On the theoretical side, there are recent calculations of these quantities using chiral perturbation theory (ChPT). In Ref. [7], γ was evaluated in a $\mathcal{O}(p^3)$ calculation in heavy baryon ChPT (HBChPT), with the result:

$$\gamma = \frac{e^2}{4\pi} \frac{g_A^2}{24\pi^2 F_\pi^2 m_\pi^2} = 4.6, \quad (1)$$

where $F_\pi = 92.4 \text{ MeV}$, $m_\pi = 137 \text{ MeV}$, and $g_A = 1.26$. This prediction is in clear disagreement with the multipole analysis. Sub-leading effects evaluated within a relativistic ChPT calculation at the one-loop level diminish the discrepancy, yielding [7] $\gamma^{1-loop} = 2.2$. Moreover, inclusion of the $\Delta(1232)$ resonance through higher order contact terms within the same approach [7] revealed large effects on γ , with opposite sign to the one-loop result, $\gamma^\Delta = -3.7$. The net result in the relativistic calculation,

$$\gamma = +2.2 (1\text{-loop}) - 3.7 (\Delta) = -1.5, \quad (2)$$

is very close to the one found with the multipole analysis mentioned above.

The individual spin polarizabilities were calculated in Ref. [8] within the same HBChPT approach as in Ref. [7]. An interesting result of this calculation is that the values of γ_1, γ_3 and γ_4 are completely dominated by the contribution of the t -channel π^0 -exchange (Wess-Zumino-Witten term), while this contribution cancels in the expression for γ . Specifically, the expressions obtained for the γ_i 's are

$$\begin{aligned}\gamma_1 &= \frac{e^2}{4\pi} \frac{g_A}{4\pi^2 F_\pi^2 m_\pi^2} \left(-1 + \frac{g_A}{6} \right), & \gamma_2 &= \frac{e^2}{4\pi} \frac{g_A}{4\pi^2 F_\pi^2 m_\pi^2} \left(0 + \frac{g_A}{12} \right), \\ \gamma_3 &= \frac{e^2}{4\pi} \frac{g_A}{4\pi^2 F_\pi^2 m_\pi^2} \left(\frac{1}{2} + \frac{g_A}{24} \right), & \gamma_4 &= -\gamma_3.\end{aligned}\tag{3}$$

Very recently, in Ref. [9] the $\Delta(1232)$ was introduced as an explicit degree of freedom in HBChPT. In this approach a new dimensionful parameter enters, $\Delta = M_\Delta - M$, and a consistent HBChPT formalism can be set up according to an $\mathcal{O}(\epsilon^n)$ power counting scheme, where ϵ denotes a small scale (a soft momentum, m_π or Δ). In this formalism the previous $\mathcal{O}(p^n)$ HBChPT results [8] are exactly reproduced, and the terms that come from treating the $\Delta(1232)$ as an explicit degree of freedom can be clearly identified. The value for γ obtained to $\mathcal{O}(\epsilon^3)$ is

$$\gamma = \frac{e^2}{4\pi} \left\{ \frac{g_A^2}{24\pi^3 F_\pi^2 m_\pi^2} - \frac{4b_1^2}{9M^2} \frac{1}{\Delta^2} - \frac{g_{\pi N\Delta}^2}{54\pi^2 F_\pi^2} \left[\frac{\Delta^2 + 2m_\pi^2}{(\Delta^2 - m_\pi^2)^2} - \frac{3m_\pi^2 \Delta}{(\Delta^2 - m_\pi^2)^{5/2}} \ln R \right] \right\}, \tag{4}$$

where $R = \Delta/m_\pi + [(\Delta/m_\pi)^2 - 1]^{1/2}$, $g_{\pi N\Delta}$ is the pion-nucleon- $\Delta(1232)$ coupling constant, and b_1 is a finite $\mathcal{O}(\epsilon^2)$ counterterm. The terms in Eq. (4) correspond, in order, to nucleon-loop (N-loop), delta-pole (Δ -pole) and delta-loop (Δ -loop). As remarked in Ref. [9], both $g_{\pi N\Delta}$ and b_1 are not well constrained by the experiment. However, in using the values $g_{\pi N\Delta} = 1.5 \pm 0.2$ and $b_1 = -(2.5 \pm 0.35)$ [10] in Eq. (4), Hemmert et al. [9] obtained

$$\gamma = 4.6 \text{ (N-loop)} - 4.0 \text{ (}\Delta\text{-pole)} - 0.4 \text{ (}\Delta\text{-loop)} = +0.2. \tag{5}$$

As in the calculation of Ref. [7], the contribution of the $\Delta(1232)$ is large and negative, but cannot account for the value $\gamma = -1.3$ found from the multipole analysis. Unfortunately, also the values for the electric and magnetic polarizabilities, α and β , are larger than the experimental data.

More recently, Holstein et al. [11] [12] have argued that the values of $g_{\pi N\Delta}$ and b_1 to be used should be determined, for consistency, within the same “small scale expansion” of the HBChPT calculation, and not within a relativistic Born model. In such a case, the values for

these quantities are approximately 50% smaller than the ones used above. The consequence of this is that α and β decrease significantly, but on the other hand, the value for γ increases significantly:

$$\gamma = 4.6 \text{ (N-loop)} - 2.4 \text{ } (\Delta - \text{pole}) - 0.2 \text{ } (\Delta - \text{loop}) = +2.0, \quad (6)$$

in contradiction with the multipole analysis of Ref. [5]. The effect of the Δ pole is also quite large for some of the γ_i 's. In Table I we show the results for the γ_i 's using for $g_{\pi N \Delta}$ and b_1 the values suggested by Holstein et al. [11] [12].

As mentioned above, there are no direct measurements of the spin polarizabilities. Nevertheless, as in the case of γ [5] [6], an estimate for the γ_i 's can be obtained using the experimentally determined amplitudes for pion photoproduction in a dispersion integral. However, there are additional contributions from t -channel processes that are not well constrained experimentally, corresponding to possibly large high-energy contributions to the dispersion integrals, similar to the effects of the Wess-Zumino-Witten term in the case of ChPT. In this sense, only the low energy part of the Compton process, i.e. contributions of the pion cloud and the low-lying resonances, can be at present constrained by a dispersion calculation.

In this Letter we present the results of a dispersion calculation for the spin polarizabilities. In order to minimize uncertainties with respect to the high-energy behavior of the Compton amplitude, we follow the approach of L'vov et al. [13]. In this approach one uses a finite-size contour in the complex plane rather than improving the convergence of the integral by subtractions. The contributions arising from the contour are expressed in terms of low-mass meson exchanges in the t -channel, as will be discussed in the next section. We use pion photoproduction multipoles from a recent analysis based on fixed t dispersion relations and unitarity by Hanstein, Drechsel and Tiator (HDT) [14]. This analysis is limited to a maximum photon laboratory energy of the order of 500 MeV. In order to investigate the sensitivity of our results to the photoproduction amplitudes at higher energies, we also employ the multipole analysis VPI-SP97K of the Scattering Analysis Interactive Dial-in program, SAID [15]. In Section 3 we present our numerical results and compare them with the results of ChPT. Some conclusions and perspectives are presented in Section 4.

2. Dispersion relations for the spin polarizabilities.

In the c.m. system the T -matrix for Compton scattering on the nucleon can be written, in the Coulomb gauge, in terms of six amplitudes that are functions of the energy of the incident photon, ω , and the scattering angle, θ , as [16]:

$$T = \chi_s^\dagger \left\{ \epsilon'^* \cdot \epsilon \bar{A}_1(\omega, \theta) + \epsilon'^* \cdot \hat{\mathbf{k}} \epsilon \cdot \hat{\mathbf{k}}' \bar{A}_2(\omega, \theta) + i \boldsymbol{\sigma} \cdot (\epsilon'^* \times \epsilon) \bar{A}_3(\omega, \theta) + i \boldsymbol{\sigma} \cdot (\hat{\mathbf{k}}' \times \hat{\mathbf{k}}) \epsilon'^* \cdot \epsilon \bar{A}_4(\omega, \theta) \right. \\ \left. + i \boldsymbol{\sigma} \cdot [(\epsilon'^* \times \hat{\mathbf{k}}) \epsilon \cdot \hat{\mathbf{k}}' - (\epsilon \times \hat{\mathbf{k}}') \epsilon'^* \cdot \hat{\mathbf{k}}] \bar{A}_5(\omega, \theta) + i \boldsymbol{\sigma} \cdot [(\epsilon'^* \times \hat{\mathbf{k}}') \epsilon \cdot \hat{\mathbf{k}} - (\epsilon \times \hat{\mathbf{k}}) \epsilon'^* \cdot \hat{\mathbf{k}}] \bar{A}_6(\omega, \theta) \right\} \chi_s, \quad (7)$$

where $\hat{\mathbf{k}}, \epsilon$ ($\hat{\mathbf{k}}', \epsilon'$) are the direction and the polarization vector of the incident (final) photon, χ_s ($\chi_{s'}$) is the initial (final) nucleon spinor, and $\boldsymbol{\sigma}$ is the Pauli spin matrix. For convenience, we write the amplitudes \bar{A}_i as a sum of the Born and non-Born contributions, $\bar{A}_i = \bar{A}_i^{\text{Born}} + \bar{A}_i^{\text{nB}}$. Ragusa's spin polarizabilities can be expressed in terms of the non-Born amplitudes \bar{A}_i^{nB} 's by [8]

$$\gamma_1 = \frac{1}{4\pi} \frac{1}{6} \frac{\partial^3}{\partial \omega^3} \left[\bar{A}_3^{\text{nB}}(\omega, 0) + \bar{A}_4^{\text{nB}}(\omega, 0) + 2\bar{A}_5^{\text{nB}}(\omega, 0) \right]_{\omega=0}, \quad \gamma_2 = \frac{1}{4\pi} \frac{1}{6} \frac{\partial^3}{\partial \omega^3} \left[\bar{A}_4^{\text{nB}}(\omega, 0) \right]_{\omega=0}, \\ \gamma_3 = \frac{1}{4\pi} \frac{1}{6} \frac{\partial^3}{\partial \omega^3} \left[\bar{A}_6^{\text{nB}}(\omega, 0) \right]_{\omega=0}, \quad \gamma_4 = \frac{1}{4\pi} \frac{1}{6} \frac{\partial^3}{\partial \omega^3} \left[\bar{A}_5^{\text{nB}}(\omega, 0) \right]_{\omega=0}. \quad (8)$$

On the other hand, the scattering amplitude can be written in terms of the Hearn-Leader (HL) [17] Lorentz-scalar amplitudes $T_i(\nu, t)$, $i = 1 \cdots 6$, as

$$T = \epsilon^\mu \bar{u}_{s'}(p') \left[\sum_{i=1}^6 I_{\mu\nu}^i T_i(\nu, t) \right] u_s(p) \epsilon^\nu, \quad (9)$$

where ϵ^μ and ϵ^ν are the polarization four-vectors of the nucleon, $\bar{u}_s(p)$ and $u_s(p)$ are Dirac spinors, and $I_{\mu\nu}^i$ are tensors that depend on Dirac γ matrices and the initial and final momenta of the photon and the proton. From arguments based on the asymptotic behavior of Regge trajectories, some of the HL amplitudes appear to have a bad convergence for fixed t and large ν . The traditional approach in dispersion theory is to implement a subtraction at threshold for these amplitudes with a bad asymptotic behavior, at the cost of introducing subtraction functions that in most cases are unknown. However, there is an additional problem with the HL amplitudes $T_i(\nu, t)$, in that they have to fulfil kinematic constraints because the tensors $I_{\mu\nu}^i$ develop singularities at forward and backward scattering angles [18]. It is possible to avoid kinematic constraints and singularities by using appropriate combinations of the amplitudes, as for example the ones of Bardeen and Tung [18]. One natural way to proceed is the one followed by Pfeil, Rollnik and Stankowski [19], who used a partial wave decomposition of the HL amplitudes. While the constraints can be automatically satisfied within this procedure,

it has the disadvantage of introducing a possible violation of the $s - u$ crossing symmetry. Another approach has been introduced by L'vov et. al [13], who expresses the dispersion contribution as the sum of a finite-range integral from threshold ν_{thr} up to a maximum value ν_{max} , and the contributions from higher energies in terms of t -channel poles. Thus, the real parts of the amplitudes are written as a sum of the real pole Born terms, the contribution of the finite-range dispersion integral, and an asymptotic t -channel contribution (we use the conventions and notations of Ref. [13]):

$$\text{Re}A_i(\nu, t) = A_i^{\text{Born}}(\nu, t) + \frac{2}{\pi} \text{P} \int_{\nu_{\text{thr}}}^{\nu_{\text{max}}} \text{Im}A_i(\nu', t) \frac{\nu' d\nu'}{\nu'^2 - \nu^2} + A_i^{\text{as}}(t). \quad (10)$$

The amplitudes $A_i(\nu, t)$ are appropriate combinations of the HL amplitudes, and are free of kinematic constraints and singularities. Two of these six amplitudes, A_1 and A_2 , appear to have a bad convergence behavior for high ν , at fixed t . The t -channel contribution to $A_1^{\text{as}}(t)$ and $A_2^{\text{as}}(t)$ are modeled by σ and π^0 exchanges, respectively, and for the remaining amplitudes the t -channel contributions seem to give negligible contribution at low energies.

In order to evaluate the spin polarizabilities in terms of the dispersion integral, one needs the relation of the amplitudes $\bar{A}_i(\omega, \theta)$ of Eq. (7) to the Lorentz-scalar amplitudes $A_i(\nu, t)$. The relation is easily obtained comparing Eq. (7) with the low energy expansion of the Compton scattering amplitude written in terms of the non-Born contributions to the Lorentz-scalars $A_i(\nu, t)$, with the result [20] [21]

$$\begin{aligned} \gamma_1 &= -\frac{1}{4\pi} \frac{1}{M} \left[A_2^{\text{nB}}(0, 0) - A_4^{\text{nB}}(0, 0) + A_5^{\text{nB}}(0, 0) \right], \\ \gamma_2 &= +\frac{1}{4\pi} \frac{1}{M} \left[A_5^{\text{nB}}(0, 0) - A_6^{\text{nB}}(0, 0) \right], \\ \gamma_3 &= +\frac{1}{8\pi} \frac{1}{M} \left[A_2^{\text{nB}}(0, 0) - A_4^{\text{nB}}(0, 0) - A_6^{\text{nB}}(0, 0) \right], \\ \gamma_4 &= -\frac{1}{8\pi} \frac{1}{M} \left[A_2^{\text{nB}}(0, 0) + A_4^{\text{nB}}(0, 0) + 2A_5^{\text{nB}}(0, 0) - A_6^{\text{nB}}(0, 0) \right]. \end{aligned} \quad (11)$$

From these expressions one sees that the polarizabilities γ_1 , γ_3 and γ_4 depend on the amplitude A_2 . As we discussed previously, this amplitude has a bad asymptotic behavior for fixed t and large ν and not well-constrained t -channel contributions. One way to avoid such uncertainties is to consider appropriate combinations of the γ_i 's to which A_2 does not contribute. In addition to the combination that leads to γ , there are more of such combinations, which will be discussed in the next section.

3. Numerical results.

In the dispersion integral Eq. (10) we saturate the imaginary parts by one-pion photo-production amplitudes. We neglect intermediate states with more pions and heavier mesons. The contributions of such states to γ have been estimated in Ref. [5] to be relatively small. As stated above, we employ two sets of photoproduction amplitudes, the ones obtained with a fixed t dispersion relations (HDT) and the ones from the SAID program. Our results using the dispersion relations (DR) for the spin polarizabilities of the proton and the neutron are presented in Tables I, II, and III.

The results in Table I are obtained by evaluating the integral in Eq. (10) with the HDT [14] multipoles up to a $\nu_{\max} = E_{\gamma}^{\max} + t/4M$, with $t = 0$ and $E_{\gamma}^{\max} = 500$ MeV. Tables II and III also show the results for the integration with the VPI-SP97K SAID multipoles up to $E_{\gamma}^{\max} = 500$ and 1500 MeV, which are denoted by SAID₁ and SAID₂, respectively. For the t -channel contribution $A_2^{\text{as}}(t)$ we use the parametrization given in Ref. [13].

In Table I we show the separate contributions from π^0 exchange and the dispersion integrals (“excitation”) to the polarizabilities. The contribution of the π^0 exchange to Eq. (10) is practically identical to the Wess-Zumino-Witten term of the HBChPT calculation. Because of the huge contribution from that term, our global result (“sum”) is quite similar to the prediction of HBChPT [12]. However, the contributions beyond the anomaly show significant differences. In Table II we present the results for these excitation contributions to the spin polarizabilities using the HDT and SAID multipoles. One notices that the HDT and SAID₁ results are fairly similar for all the γ_i ’s. On the other hand, extending the upper limit of the integration of the SAID multipoles from 500 MeV to 1500 MeV changes the results for $\gamma_1^{(p)}$, $\gamma_2^{(p)}$, and $\gamma_4^{(p)}$ by a factor of the order of 25%, and changes the sign and magnitude of $\gamma_3^{(p)}$. The values of the $\gamma_i^{(n)}$ ’s are more stable with respect to the change of the upper limit of the integration. We come back to this point shortly ahead. As already seen in Table I, the results from DR are at variance with the predictions of HBChPT in most cases.

As we discussed above, there exist combinations of the γ_i ’s that do not depend on the amplitude A_2 and thus are not affected by the badly known high-energy contributions. One of these combinations is $\gamma = \gamma_1 - \gamma_2 - 2\gamma_4$, and three other combinations are:

$$\gamma_{13} = \gamma_1 + 2\gamma_3, \quad \gamma_{14} = \gamma_1 - 2\gamma_4, \quad \gamma_{34} = \gamma_3 + \gamma_4. \quad (12)$$

Note that only two of these combinations are independent, since $\gamma_{13} - \gamma_{14} = 2\gamma_{34}$. The results for γ , γ_{13} , and γ_{14} are presented in Table III.

Our results for $\gamma^{(p)}$ and $\gamma^{(n)}$ using the SAID multipoles up to 1500 MeV are practically identical to the values obtained for these quantities in Ref. [5] using the VPI-FA93 analysis (see Ref. [5] for details). We note that the values of $\gamma^{(p)}$ and $\gamma^{(n)}$ using the VPI-SP97K SAID multipoles up to 500 MeV and 1500 MeV differ by less than 10%, i.e. there are no large contributions to the dispersion integrals from energies between 500 MeV and 1500 MeV. This fact can be understood as due to the damping factor $1/\omega^3$ in the integrand of the expression for γ ,

$$\gamma = \int_{\omega_{\text{thr}}}^{\infty} \frac{\sigma_{1/2} - \sigma_{3/2}}{4\pi^2\omega^3} d\omega, \quad (13)$$

where $\sigma_{1/2}$ and $\sigma_{3/2}$ are the total photoabsorption cross sections measured with the photon and nucleon polarizations parallel and antiparallel, respectively. On the other hand, we find that the value of γ calculated with the HDT multipoles is a factor of two smaller than the value calculated with the SAID multipoles. This can be traced to differences between the E_{0+} amplitudes in both analyses. In Figs. 1 (a) and (b) we plot the contributions of the E_{0+} and M_{1+} multipoles (which are by far the largest contributors) to the integrand in Eq. (13), and in Fig. 1(c) we plot the total integrand. From these figures it becomes obvious that the difference comes from the behavior of the E_{0+} multipoles close to threshold. As remarked in Ref. [22], the SAID multipoles are not meant to be used in the π^+n threshold region. For example, the amplitude $E_{0+}(n\pi^+)$ at threshold is $24.9 \times 10^{-3}/m_{\pi^+}$ for SAID and $28.3 \times 10^{-3}/m_{\pi^+}$ for HDT, the latter being much closer to the threshold value predicted by ChPT [23], $28.4 \times 10^{-3}/m_{\pi^+}$. The differences between the E_{0+} amplitudes of the HDT and SAID multipoles is of the order of 40% at the peak value of the integrand of γ . As seen in Fig. 1, this large difference between the E_{0+} amplitudes, and a small difference between the M_{1+} amplitudes (the SAID value is a little larger than the HDT value), give a net effect of 50% in the final value for γ .

An interesting comparison can be made between the DR and HBChPT results by considering in separate the leading contributions to γ , which come from the E_{0+} , E_{1+} , and M_{1+} multipoles. Using the HDT multipoles, we obtain:

$$\gamma = +2.5 (E_{0+}) - 3.0 (E_{1+}, M_{1+}) - 0.1 (\text{rest}) = -0.6, \quad (14)$$

where the last term includes all other partial waves. This shows the strong cancelation between s-wave pion loop and Δ resonance contributions, which is in close correspondence with a similarly strong cancelation in the ChPT calculations, Eqs. (2), (5), and (6). It is

amusing to note that the first two numbers in Eq. (14) are not very different from the ones in Eq. (2) obtained in the relativistic ChPT calculation of Ref. [7], namely $\gamma^{1-loop} = 2.2$ and $\gamma^\Delta = -3.7$.

To conclude our discussion on the numerical results, we note that the results in Table III clearly show that when the troublesome amplitude $A_2(\nu, t)$ is eliminated by taking appropriate combinations of the γ_i 's, the results become very stable with respect to the upper limit of the integral. In fact, similar to the case with γ , the integrals for the other combinations of γ_i 's are almost completely saturated with an upper limit of 500 MeV. The large changes in the SAID₁ and SAID₂ predictions observed in Table II can therefore be traced to the presence of the function $A_2(\nu, t)$ in the expressions of the γ_i 's. In fact, inspection of the integrands for the γ_i 's reveals [21] that these still receive sizable contributions from high energies due to the function $A_2(\nu, t)$.

4. Conclusions and Perspectives.

In this paper we use dispersion relations to estimate the values of the spin polarizabilities γ_i of the nucleon and compare our results with HBChPT calculations. By taking appropriate combinations of the γ_i 's, we are able to minimize uncertainties related to t -channel contributions. The present dispersion calculation predicts, in accord with previous calculations [5] [6], an opposite sign for $\gamma = \gamma_1 - \gamma_2 - 2\gamma_4$ as compared to the HBChPT calculation. The same discrepancy is seen for $\gamma_{14} = \gamma_1 - 2\gamma_4$. Note that in the HBChPT calculation [12], γ_1 receives sizable contributions from $\pi - N$ loops but no Δ -pole contributions, whereas in γ_2 and γ_4 there are cancelations between contributions from $\pi - N$ loops and the Δ -pole graphs [12]. Therefore the required change of sign in both γ and γ_{14} seems nontrivial and highly constrained. In this sense, it would be extremely interesting to see the outcome of a systematic HBChPT calculation of these quantities to $\mathcal{O}(\epsilon^4)$.

Another interesting conclusion of our calculation is that the spin polarizabilities are very sensitive to the behavior of the photoproduction multipole E_{0+} near threshold. There is a large cancelation between the contributions from the E_{0+} and M_{1+} multipoles, and a very precise determination of both of these is necessary to constrain the values of the spin polarizabilities.

We conclude by remarking that similar to the electric and magnetic polarizabilities α and β , the spin polarizabilities $\gamma_1 \cdots \gamma_4$ contain vital information on the low energy structure of the nucleon. The experimental determination of these new polarizabilities is therefore of great interest.

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TABLES

TABLE I. Separate contributions to the spin polarizabilities from the HBChPT calculation of Ref. [12] and the result from the dispersion calculation using the HDT [14] multipoles (all results are in units of 10^{-4} fm^4).

γ_i	HBChPT			DR (HDT)		
	WZW	excitation	sum	π^0 -exchange	excitation	sum
$\gamma_1^{(p)}$	-22.0	+4.4	-17.6	-22.5	+5.1	-17.4
$\gamma_2^{(p)}$	0	-0.3	-0.3	0	-1.1	-1.1
$\gamma_3^{(p)}$	+11.0	+1.1	+12.1	+11.2	-0.6	+10.6
$\gamma_4^{(p)}$	-11.0	+1.3	-9.7	-11.2	+3.4	-7.9
$\gamma_1^{(n)}$	+22.0	+4.4	+26.4	+22.5	+6.1	+28.6
$\gamma_2^{(n)}$	0	-0.3	-0.3	0	-0.8	-0.8
$\gamma_3^{(n)}$	-11.0	+1.1	-9.9	-11.2	-0.6	-11.8
$\gamma_4^{(n)}$	+11.0	+1.3	+12.3	+11.2	+3.4	+14.6

TABLE II. Excitation contribution to the spin polarizabilities from the HBChPT calculation of Ref. [10] and the result from the dispersion calculation using the HDT, SAID₁, and SAID₂ multipoles (all results are in units of 10^{-4} fm^4).

$\gamma_i - \text{excit.}$	HBChPT	DR (HDT)	DR (SAID ₁)	DR (SAID ₂)
$\gamma_1^{(p)}$	+4.4	+5.1	+4.3	+3.5
$\gamma_2^{(p)}$	-0.3	-1.1	-1.2	-1.0
$\gamma_3^{(p)}$	+1.1	-0.6	-0.5	+0.1
$\gamma_4^{(p)}$	+1.3	+3.4	+3.4	+2.9
$\gamma_1^{(n)}$	+4.4	+6.1	+5.9	+6.1
$\gamma_2^{(n)}$	-0.3	-0.8	-1.0	-0.9
$\gamma_3^{(n)}$	+1.1	-0.6	-0.6	-0.6
$\gamma_4^{(n)}$	+1.3	+3.4	+3.5	+3.6

TABLE III. Combinations of spin polarizabilities that do not depend on the amplitude $A_2(\nu, t)$ (all results are in units of 10^{-4} fm^4).

$\gamma's$	HBCbPT	DR (HDT)	SAID ₁	SAID ₂
$\gamma^{(p)}$	+2.0	-0.6	-1.2	-1.3
$\gamma_{13}^{(p)}$	+6.6	+3.8	+3.3	+3.7
$\gamma_{14}^{(p)}$	+1.8	-1.7	-2.4	-2.3
$\gamma^{(n)}$	+2.0	+0.0	-0.2	-0.3
$\gamma_{13}^{(n)}$	+6.6	+4.9	+4.7	+4.9
$\gamma_{14}^{(n)}$	+1.8	-0.7	-1.2	-1.1

FIGURES

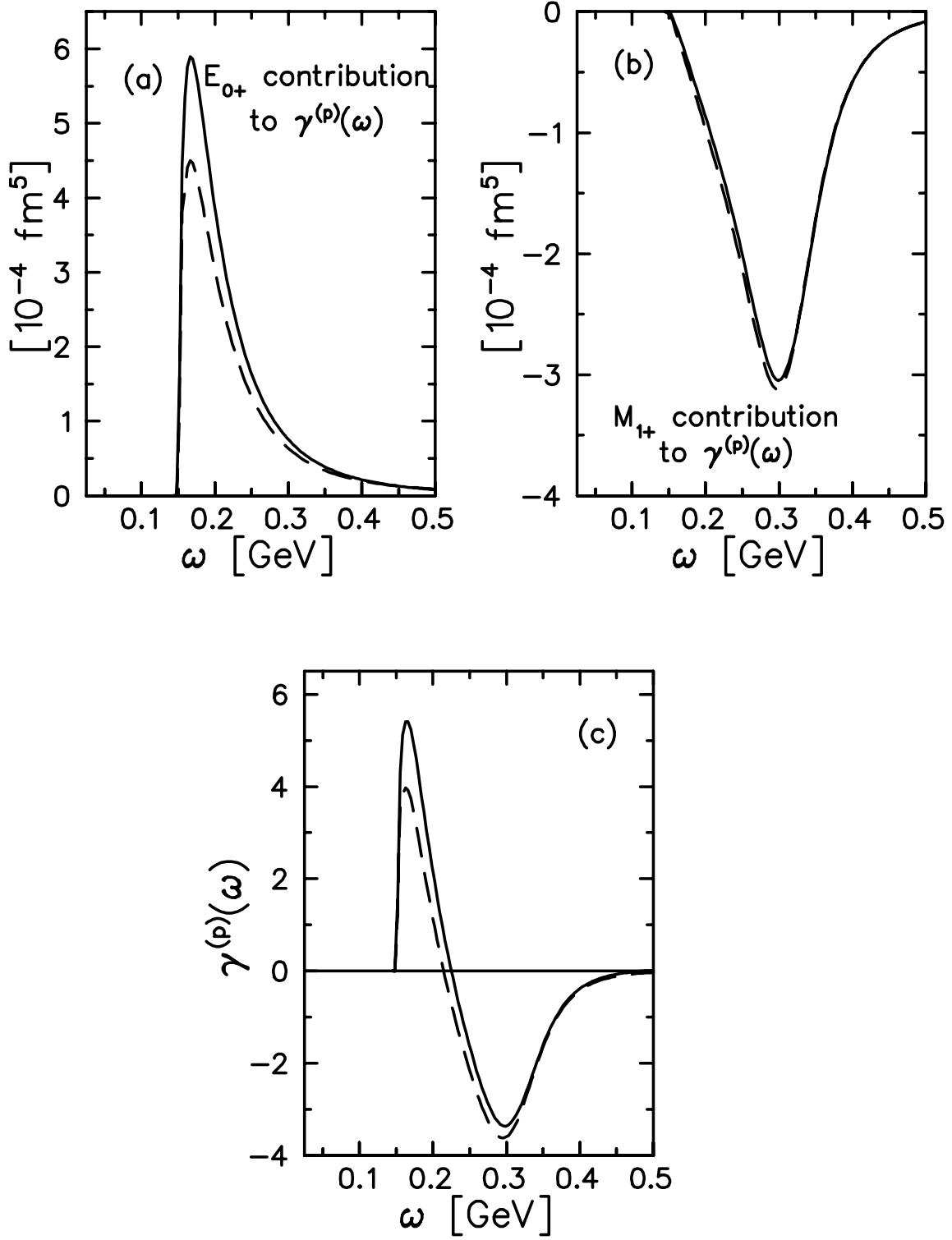


FIG. 1. Contribution of the multipoles E_{0+} (a) and M_{1+} (b) to the integrand for $\gamma^{(p)}$ (c) from the HDT (solid line) and the SAID (dashed line) analyses.